

Controllability issues for continuous-spectrum systems and ensemble controllability of Bloch equations

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We study the controllability of the Bloch equation, for an ensemble of non interacting spin $1/2$, in a static magnetic field, with dispersion in the Larmor frequency. This system may be seen as a prototype for infinite dimensional bilinear systems with continuous spectrum, whose controllability is not well understood. We provide several mathematical answers, with discrimination between approximate and exact controllability, and between finite time or infinite time controllability: this system is not exactly controllable in finite time T with bounded controls in $L^2(0, T)$, but it is approximately controllable in L^2 in finite time T with unbounded controls in $L^\infty(0, T)$; moreover, it is exactly controllable in infinite time, with unbounded controls in $L_{loc}^\infty(0, \infty)$.